3.3: Higher-Order Linear, Homogeneous Equations with Constant Coefficients
Real Roots

Theorem 1. (Distinct Real Roots)
If the roots $r_1, r_2, \ldots, r_n$ of the characteristic equation (2) are real and distinct, then

$$y(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + \cdots + c_n e^{r_n x}$$

is a general solution to (1).

Exercise 1. Solve the initial value problem

$$y^{(3)} + 3y'' - 10y' = 0, \quad y(0) = 7, y'(0) = 0, y''(0) = 70.$$

Theorem 2. (Repeated Roots)
If the characteristic equation (2) has a repeated real root r of multiplicity k, then the part of a general solution of the differential equation (1) corresponding to r is of the form

$$(c_1 + c_2 x + \cdots + c_{k-1} x^{k-2} + c^k x^{k-1}) e^{rx}.$$

Exercise 2. Find a general solution to the equation

$$9y^{(5)} - 6y^{(4)} + y^{(3)} = 0.$$